



Spatial chaotic vibrations when there is a periodic change in the position of the centre of mass of a body in the atmosphere[☆]

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ABSTRACT

The spatial chaotic motion of a blunt body in the atmosphere when there is a periodic change in the position of the centre of mass is considered. A restoring moment, described by a biharmonic dependence on the spatial angle of attack, a small perturbing moment, due to the periodic change in the position of the centre of mass, and also a small damping moment, acts on the body. The motion when the velocity head remains constant is investigated. When there are no small perturbations, the phase portrait of the system can have points of stable and unstable equilibrium. The behaviour of the system in the neighbourhood of the separatrix is investigated using Mel'nikov's method. An analytic solution of the equation of the body motion along the separatrix is obtained. The criteria for the occurrence of chaos are obtained and the results of numerical modelling, which confirm the correctness of the solutions obtained, are presented.

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When investigating the motion of a rigid body around the centre of mass when it descends through the atmosphere, the most complex problems arise when investigating resonance, which has a considerable effect on the behaviour of the body (see Refs 1,2, for example). The classical methods of non-linear mechanics^{3,4} are usually employed when solving this problem. However, methods of chaotic dynamics, in particular, Mel'nikov's method,^{5–10} have become more and more widely used.

1. Formulation of the problem

We will consider a body of axisymmetric shape, the aerodynamic characteristics of which, as a rule, are specified using the dependences of the tangential force coefficient $c_t(\alpha)$, the normal force coefficient $c_n(\alpha)$, the position of the centre of pressure $x_d(\alpha)$ on the spatial angle of attack α .¹¹ The coefficient of the moment about the nose of the body $m_0(\alpha)$ is often used instead of the coordinate of the centre of pressure. The coefficient of the static aerodynamic moment about the centre of mass of the body is then given by the formula

$$m_\alpha(\alpha) = -c_n(\alpha)[\bar{x}_d(\alpha) - \bar{x}_c] = m_0(\alpha) + c_n(\alpha)\bar{x}_c; \quad \bar{x}_d = x_d/L, \quad \bar{x}_c = x_c/L \quad (1.1)$$

where x_c is the coordinate of the centre of mass of the body about the nose and L is the characteristic length of the body. The static aerodynamic moment has the form

$$M_\alpha = m_\alpha(\alpha)qSL \quad (1.2)$$

where S is the area of the midsection of the body and q is the velocity head (the dynamic pressure). In addition to the static moment (1.2) a small damping moment

$$M_D = -\delta m^\omega(\alpha)\dot{\alpha} \quad (1.3)$$

acts on the body, where δ is small positive parameter and m^ω is an even periodic function of the angle of attack; for a spherical body $m^\omega \approx 1 + \sin^2\alpha$.¹ We will use this relation below.

Uncontrolled short spacecraft, having a blunt shape^{11,12} are used for effective braking in the rarefied atmosphere of Mars. Such spacecraft, in addition to having two balancing positions of the spatial angle of attack ($\alpha^* = 0, \pi$), may also have a third position of equilibrium $\alpha^* \in (0, \pi)$,

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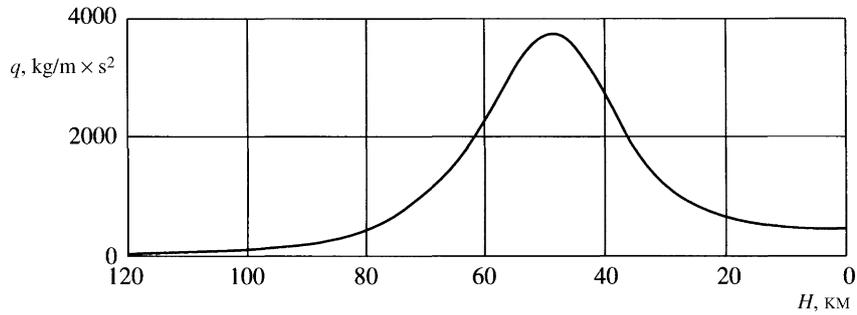


Fig. 1.

depending on the position of the centre of mass. To approximate the static moment (1.2), a biharmonic dependence on the spatial angle of attack² is used, namely,

$$M_{\alpha}(\alpha) = a \sin \alpha + b \sin 2\alpha \tag{1.4}$$

A spacecraft of this class in the position of equilibrium $\alpha = 0$ must be statically stable, and hence

$$\left. \frac{dM_{\alpha}(\alpha)}{d\alpha} \right|_{\alpha=0} = (a \cos \alpha + 2b \cos 2\alpha)|_{\alpha=0} = a + 2b < 0 \tag{1.5}$$

If an intermediate balancing position exists in the range $(0, \pi)$, the following equality holds

$$M_{\alpha}(\alpha) = a \sin \alpha + b \sin 2\alpha = \sin \alpha (a + 2b \cos \alpha) = 0$$

This is satisfied when

$$|b| > |a|/2 \tag{1.6}$$

When $b < 0$ inequalities (1.5) and (1.6) are satisfied simultaneously.

In Fig. 1 we show a graph of the velocity head against the height for the descent trajectory of the Beagle 2 spacecraft¹³ for the following initial conditions of entry into the Mars atmosphere: height $H_0 = 120000$ m, velocity $V_0 = 3500$ m/s and entry angle $\theta_0 = -8^\circ$. At the end of the trajectory, when $H < 20000$ m a section of equilibrium descent is observed, when the velocity head q hardly changes. Along this section the soft-landing parachute system opens.

It is obvious that if the spatial angle of attack α is greater than $\pi/2$ at the instant when the parachute is uncovered, the parachute, situated on the rear part of the spacecraft, will not open. Consequently, an investigation of the behaviour of a spacecraft when it is in motion around the centre of mass on the final part of the trajectory is extremely important. Various perturbations, related to the low aerodynamic and dynamic asymmetry, act on a spacecraft when it moves through the atmosphere.^{1,2} These periodic perturbations may have a considerable effect on the motion of the spacecraft.

We will consider the effect of one model form of perturbation, namely, a periodic change in the position of the centre of mass with small amplitude

$$\bar{x}_c = \bar{x}_{c0} + \Delta \bar{x}_c \sin \omega t \tag{1.7}$$

where \bar{x}_{c0} is the initial position of the centre of mass, $\Delta \bar{x}_c$ is a small positive parameter and $\omega > 0$ is the frequency of the external perturbing moment. Taking approximation (1.4) and formulae (1.1) and (1.7) into account, we will represent the aerodynamic moment (1.2) in the form

$$M_{\alpha} = a \sin \alpha + b \sin 2\alpha + \varepsilon (a \sin \alpha + b \sin 2\alpha) \sin \omega t \tag{1.8}$$

where ε is a small parameter, which satisfies the equality

$$\varepsilon (a \sin \alpha + b \sin 2\alpha) = \Delta \bar{x}_c c_n(\alpha) q S L$$

The equation of perturbed spatial motion of the body around the centre of mass can be written in the form²

$$\begin{aligned} \ddot{\alpha} + \frac{(G - R \cos \alpha)(R - G \cos \alpha)}{\sin^3 \alpha} - a \sin \alpha - b \sin 2\alpha = \\ = \varepsilon (a \sin \alpha + b \sin 2\alpha) \sin(\omega t) - \delta m^{\omega}(\alpha) \dot{\alpha} \end{aligned} \tag{1.9}$$

where R and G are the projections of the angular momentum vector onto the longitudinal axis and onto the direction of the velocity, apart from a factor.

Our problem is as follows: it is required to show that it is possible for chaos to occur in the behaviour of perturbed system (1.9) in the neighbourhood of the separatrix, and to obtain, using Mel'nikov's method,⁵ the criteria for chaos to occur.

2. Unperturbed motion

The equation of perturbed motion (1.9) when $\varepsilon=0$ and $\delta=0$ corresponds to an unperturbed system with one degree of freedom. It is obvious that when $b=0$ the homogeneous equation corresponding to Eq. (1.9) describes the motion of a solid around a fixed point in the Lagrange case.¹⁴

We will obtain the conditions for which the unperturbed system has three equilibrium positions. The corresponding homogeneous equation has an energy integral, which, after the replacement of variables $u = \cos\alpha$, can be written in the form

$$\frac{\dot{u}^2}{2(1-u^2)} + W_g(u) + W_r(u) = E \quad (2.1)$$

where

$$W_g(u) = \frac{G^2 + R^2 - 2GRu}{2(1-u^2)}, \quad W_r(u) = au + bu^2$$

We will investigate the behaviour of the function $W(u) = W_g(u) + W_r(u)$ for different combinations of the parameters R , G , a and b , on which the phase portrait of the unperturbed system depends.

The derivative of the function $W_g(u)$ with respect to the variable u

$$W'_g(u) = [(R^2 + G^2)u - RG(1 + u^2)](1 - u^2)^{-2}$$

is equal to the product of two factors. The first of these has real mutually inverse roots R/G and G/R , of which only one belongs to the section $[-1, 1]$ considered. Consequently, a unique extremum of the function $W_g(u)$ exists, where this extremum, equal to $\max(R^2, G^2)/2 \geq 0$, is obviously a minimum. Analysing the second derivative

$$W''_g(u) = [(R^2 + G^2)(1 + 3u^2) - 2RGu(3 + u^2)](1 - u^2)^{-3}$$

it can be established that it, like the function $W_g(u)$ itself, is non-negative everywhere in the section $[-1, 1]$. In fact, the first factor has extrema at the already known points R/G and G/R , equal to $(G^2 - R^2)^2/R^2 \geq 0$ and $(G^2 - R^2)^2/G^2 \geq 0$ respectively, while at the ends of the section $u = \pm 1$ it has the values $4(G \mp R)^2 \geq 0$. Hence it follows that the function $W_g(u)$ has no points of inflection, and its derivative increases monotonically over the whole section.

We will now consider the quadratic function $W_r(u)$. It has an extremum at the point $a/(2b)$, where its derivative $W'_r(u) = a + 2bu$ vanishes. The second derivative $W''_r(u) = 2b$ is a constant quantity. It follows from this that when the condition

$$b \geq - \min_{-1 \leq u \leq 1} [W''_g(u)/2] \equiv b^* \quad (2.2)$$

is satisfied, the function $W(u)$ has no points of inflection in the interval considered. This means that there is a unique stable equilibrium position on the phase portrait of the system, and there is no singular saddle point.

There will also be no saddle point if

$$|b| \leq |a|/2 \quad (2.3)$$

In this case $W''_r(u)$ has the same sign over the whole section and consequently $W'(u)=0$ at a single point, and the function $W(u)$ has a unique extremum – a minimum.

If none of conditions (2.2) and (2.3) is satisfied, the presence of two minima and a single maximum of the function $W(u)$ in the section $[-1, 1]$ is possible, which corresponds to the presence on the phase portrait of an unstable singular saddle-type point. This situation will occur when the following condition is satisfied

$$W'(u_{*1})W'(u_{*2}) < 0 \quad (2.4)$$

where u_{*1} , u_{*2} are the roots of the equation $W''(u)=0$. When condition (2.4) is satisfied the phase plane is split by the separatrix into three regions: an outer region A_0 and two inner regions A_1 and A_2 .

3. Homoclinic orbits

We will investigate two homoclinic trajectories – separatrices, belonging to the regions A_1 and A_2 , which intersect in the saddle $u = u_0$. To obtain the criterion for chaos to occur in the neighbourhood of separatrices using Mel'nikov's method it is necessary to find analytic solutions of the equation of unperturbed motion (the homogeneous equation corresponding to Eq. (1.9)) for homoclinic orbits.

We resolve the energy integral (2.1) in terms of the derivative

$$\begin{aligned} \dot{u}^2 &= 2(1-u^2)(E - au - bu^2) + 2GRu - G^2 - R^2 = \\ &= 2bu^4 + 2au^3 - 2(b+E)u^2 - 2(a-GR)u + (2E - G^2 - R^2) = f(u) \end{aligned} \quad (3.1)$$

The fourth-degree polynomial $f(u)$ has a limited number of characteristic versions of the position of the roots.² At the points $u = \pm 1$ we have

$$f(\pm 1) = -(G \mp R)^2 \leq 0 \quad (3.2)$$

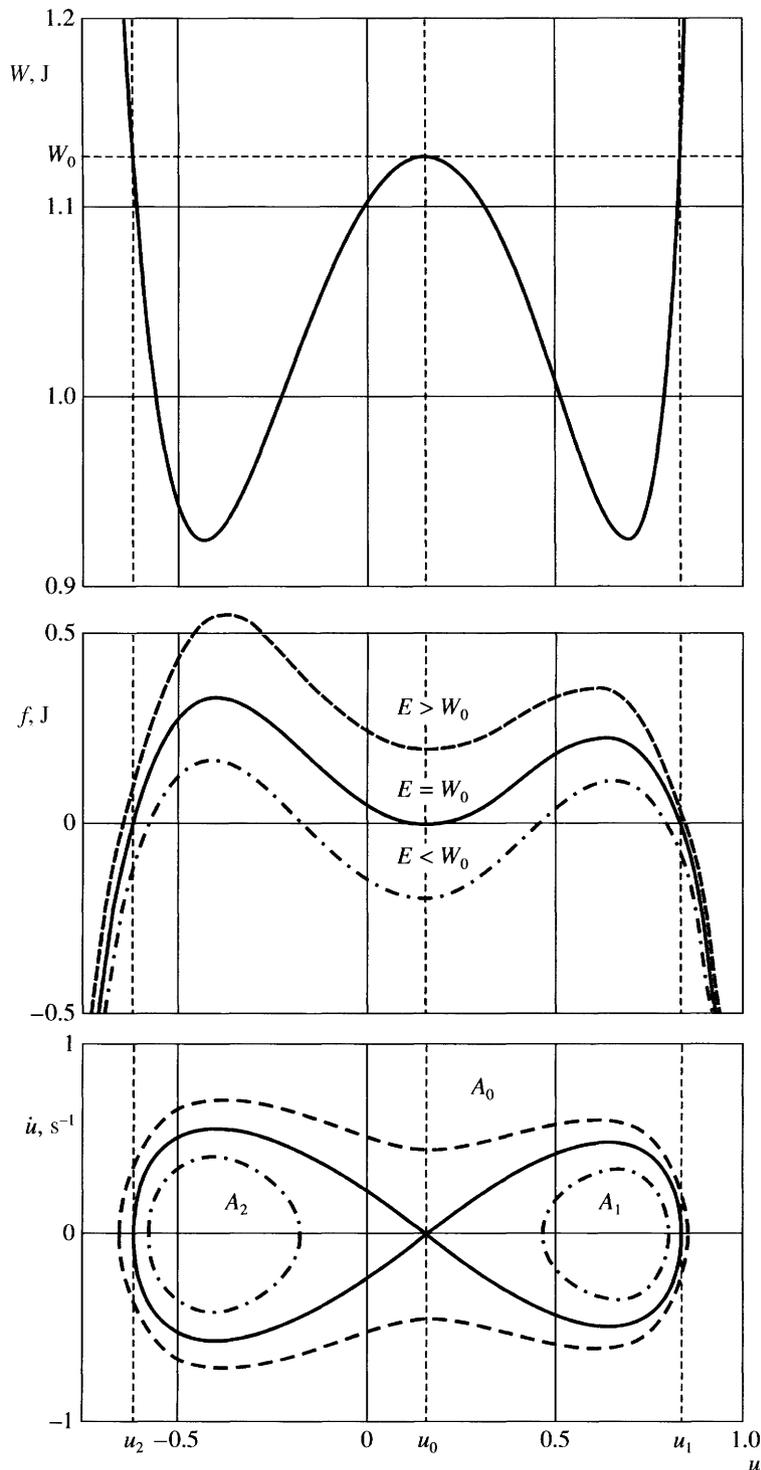


Fig. 2.

The values $u = \cos\alpha$ from the section $[-1, 1]$ and non-negative values of the function $f(u)$ correspond to the actual process, according to Eq. (3.1). By virtue of relation (3.2) the polynomial $f(u)$ has an even number of real roots in the section $[-1, 1]$. If $E > W_0$, where W_0 is the value of $W(u)$ at the saddle point, motion occurs in the outer region A_0 and the polynomial $f(u)$ has two real roots. When $e < W_0$ the polynomial $f(u)$ has four real roots and motion can occur in any of the inner regions A_1 or A_2 depending on the initial conditions. The equality $E = W_0$ corresponds to motion along the separatrix; in this case the polynomial also has four real roots, but the two inner roots are equal to one another at the saddle point $u = u_0$ (Fig. 2).

We will obtain the homoclinic trajectories of the unperturbed system. We will write Eq. (3.1) for the separatrix

$$\dot{u}^2 = -2b(u - u_0)^2(u_1 - u)(u - u_2) = f(u) \tag{3.3}$$

where u_1, u_0, u_0 and u_2 are the roots of the polynomial $f(u)$. Separating the variables in Eq. (3.3) and integrating, after making the replacement $u = x + u_0$ we obtain the well-known integral¹⁵ (D is an arbitrary constant)

$$\sqrt{-2bt} = -\frac{1}{\sqrt{A}} \ln \frac{2A + Bx + 2\sqrt{AR(x)}}{x} + D \tag{3.4}$$

where

$$A = -(u_1 - u_0)(u_2 - u_0) > 0, \quad B = u_1 + u_2 - 2u_0, \quad R(x) = A + Bx - x^2$$

Solving Eq. (3.4) for $u = x + u_0$, we obtain

$$\cos \alpha = F_i(t); \quad F_i(t) = u_0 - \frac{4A}{2B - (4A + B^2)C_i^{-1} \exp(\lambda t) - C_i \exp(-\lambda t)} \tag{3.5}$$

where $\lambda = \sqrt{2b(u_1 - u_0)(u_2 - u_0)}$ and C_i is an arbitrary constant, which is determined separately for each region A_i (Fig. 2). Here and everywhere henceforth $i = 1, 2$. The following initial conditions

$$t = 0: \alpha_0 = \arccos u_i, \quad \dot{\alpha}_0 = 0$$

give the general formula for the arbitrary constant in solution (3.5) for the regions A_i

$$C_i = \frac{2A + B(u_i - u_0) + 2\sqrt{A[A + B(u_i - u_0) - (u_i - u_0)^2]}}{u_i - u_0}$$

Finally, we will write the homoclinic trajectories in a form which is more convenient for using Mel'nikov's method

$$\alpha_{\pm}^{(i)}(t) = \pm \arccos F_i(t), \quad \sigma_{\pm}^{(i)}(t) = \pm \frac{d}{dt} \alpha_{\pm}^{(i)}(t) = \pm \frac{d}{dt} \arccos F_i(t) \tag{3.6}$$

4. Perturbed motion

Mel'nikov's criterion. We will represent the perturbed second-order non-autonomous system (1.9) in the form of a third-order autonomous system

$$\dot{\alpha} = \sigma = f_1 + g_1, \quad \dot{\sigma} = f_2 + g_2, \quad \dot{\phi} = \omega \tag{4.1}$$

where

$$f_1 = \sigma, \quad g_1 = 0$$

$$f_2 = -(G - R \cos \alpha)(R - G \cos \alpha) \sin^{-3} \alpha + a \sin \alpha + b \sin 2\alpha$$

$$g_2 = \varepsilon(a \sin \alpha + b \sin 2\alpha) \sin \phi - \delta(1 + \sin^2 \alpha) \sigma$$

Mel'nikov's function for the perturbed system (4.1) takes the form⁶

$$\begin{aligned} M_{\pm}^{(i)}(t_0, \phi_0) &= \int_{-\infty}^{\infty} \{f_1[q_{\pm}^{(i)}(t)]g_2[q_{\pm}^{(i)}(t), \omega t + \omega t_0 + \phi_0] - \\ &- f_2[q_{\pm}^{(i)}(t)]g_1[q_{\pm}^{(i)}(t), \omega t + \omega t_0 + \phi_0]\} dt = \\ &= \int_{-\infty}^{\infty} \{f_1[q_{\pm}^{(i)}(t)]g_2[q_{\pm}^{(i)}(t), \omega t + \omega t_0 + \phi_0]\} dt \end{aligned} \tag{4.2}$$

where $q_{\pm}^{(i)}(t) = [\alpha_{\pm}^{(i)}(t), \sigma_{\pm}^{(i)}(t)]$ are the solutions for the homoclinic orbits (3.6) for regions A_i . Substituting expressions (4.1) into relations (4.2), taking equalities (3.6) into account, we obtain

$$M_{\pm}^{(i)}(t_0, \phi_0) = M_{\varepsilon}^{(0)} + M_{\delta}^{(i)}$$

where

$$M_{\varepsilon}^{(i)} = \varepsilon \tilde{I}_{\pm}^{(i)}, \quad \tilde{I}_{\pm}^{(i)} = \int_{-\infty}^{\infty} \sigma_{\pm}^{(i)}(a \sin \alpha_{\pm}^{(i)} + b \sin 2\alpha_{\pm}^{(i)}) \sin(\omega t + \omega t_0 + \phi_0) dt \tag{4.3}$$

$$M_{\delta}^{(i)} = -\delta J_{\pm}^{(i)}, \quad J_{\pm}^{(i)} = \int_{-\infty}^{\infty} (1 + \sin^2 \alpha_{\pm}^{(i)}) (\sigma_{\pm}^{(i)})^2 dt \quad (4.4)$$

The functions M_{ε} and M_{δ} correspond to two forms of small perturbations: a periodic perturbing moment and a damping moment. According to Mel'nikov's method⁵ the conditions for the separatrices to intersect can be written as

$$M_{\delta}^{(i)} < M_{\varepsilon}^{(i)} \quad (4.5)$$

For the two regions A_i considered we can represent functions (4.3) in the form

$$M_{\varepsilon}^{(i)} = \varepsilon I_{\pm}^{(i)} \cos(\omega t_0 + \phi_0); \quad I_{\pm}^{(i)} = \tilde{I}_{\pm}^{(i)} \Big|_{t_0=0, \phi_0=0} \quad (4.6)$$

We will obtain the values of the improper integrals $I_{\pm}^{(i)}, J_{\pm}^{(i)}$ numerically, taking solutions (3.6) into account. We introduce the reduced damping moment coefficient $\Delta = \delta/\varepsilon$, in which case we can write condition (4.5) for the separatrices to intersect as follows, by virtue of relations (4.6)

$$\Delta < \frac{|I_{\pm}^{(1)}|}{J_{\pm}^{(1)}} = \Delta_1 \quad \text{for region } A_1, \quad \Delta < \frac{|I_{\pm}^{(2)}|}{J_{\pm}^{(2)}} = \Delta_2 \quad \text{for region } A_2 \quad (4.7)$$

By virtue of solutions (3.6) and the form of the integrals $I_{\pm}^{(i)}, J_{\pm}^{(i)}$ the coefficients Δ_1 and Δ_2 are functions of the parameters of the unperturbed system and the oscillation frequency of the centre of mass of the body ω

$$\Delta_i = \Delta_i(a, b, G, R, \omega) \quad (4.8)$$

Criteria (4.8) define the behaviour of perturbed system (1.9) or (4.1) in the neighbourhood of a separatrix.

5. Modelling of the chaotic motion

We will investigate the behaviour of perturbed system (4.1) in the neighbourhood of a separatrix by numerical integration using the Runge-Kutta method. In all the calculations the biharmonic moment coefficients (1.4) and the projections of the angular momentum are assumed to be as follows:

$$a = 1, \quad b = -2, \quad G = -1.4, \quad R = 0.5 \quad (5.1)$$

The roots of the equation $f(u)=0$ (see (3.1)), corresponding to the motion along a separatrix for parameters (5.1) and a value of the total energy $E=1.128$, are

$$u_1 = 0.8282, \quad u_0 = 0.1490, \quad u_2 = -0.6261 \quad (5.2)$$

In Fig. 3 we show Poincaré sections, constructed for instants of time when the coordinate φ is a multiple of 2π , for the cases of unperturbed motion ($\omega=0$, Fig. 3a) and perturbed motion ($\varepsilon=0.01, \omega=1$, Fig. 3b). The stable and unstable manifolds of the saddle points, which form the separatrix of the unperturbed motion (Fig. 3a), are split with the formation of intersections. This leads to chaotic behaviour of the phase trajectories in their neighbourhood (Fig. 3b).

In Fig. 4 we show the effect of the perturbing moment

$$m_p = \varepsilon(a \sin \alpha + b \sin 2\alpha) \sin(\omega t)$$

at a frequency $\omega=1$, small parameters $\varepsilon=0.01, \delta=0$ (the upper part of Fig. 4) and $\delta=0.01$ (the lower part), for the following initial conditions: $\alpha_0 = \arccos(u_2) - 0.01, \dot{\alpha}_0 = 0$ and $\phi_0 = 3\pi/2$. When $\delta=0$ the perturbed trajectory (the continuous curve) begins inside region A_2 , and then repeatedly intersects the unperturbed separatrix transferring from the inner region A_2 to the outer region A_0 and vice versa. When there is a small damping moment $\delta=0.01$, all the remaining parameters and conditions remain the same, the phase portrait is essentially changed: the phase trajectories leave region A_2 and are attracted to the corresponding centre.

In order to check the criteria we will investigate the behaviour of perturbed system (4.1) in the neighbourhood of a separatrix. For the parameters of system (5.1) and (5.2) and a frequency $\omega=1$, the critical values of Δ_1 and Δ_2 under conditions (4.7) are

$$\Delta_1 = 0.7178, \quad \Delta_2 = 1.4437$$

The following critical values of the damping moment coefficient correspond to them

$$\delta_1^* = \varepsilon \Delta_1 = 0.00718, \quad \delta_2^* = \varepsilon \Delta_2 = 0.01444 \quad (5.3)$$

Hence it follows that perturbed motion in the inner region A_2 is more liable to chaos than motion in the inner region A_1 .

In Fig. 5 we show the effect of the damping moment coefficient on the behaviour of the perturbed system for initial conditions in the region of the separatrix

$$\alpha_0 = \arccos u_2 - 0.002, \quad \dot{\alpha}_0 = 0, \quad \phi_0 = 5\pi/4$$

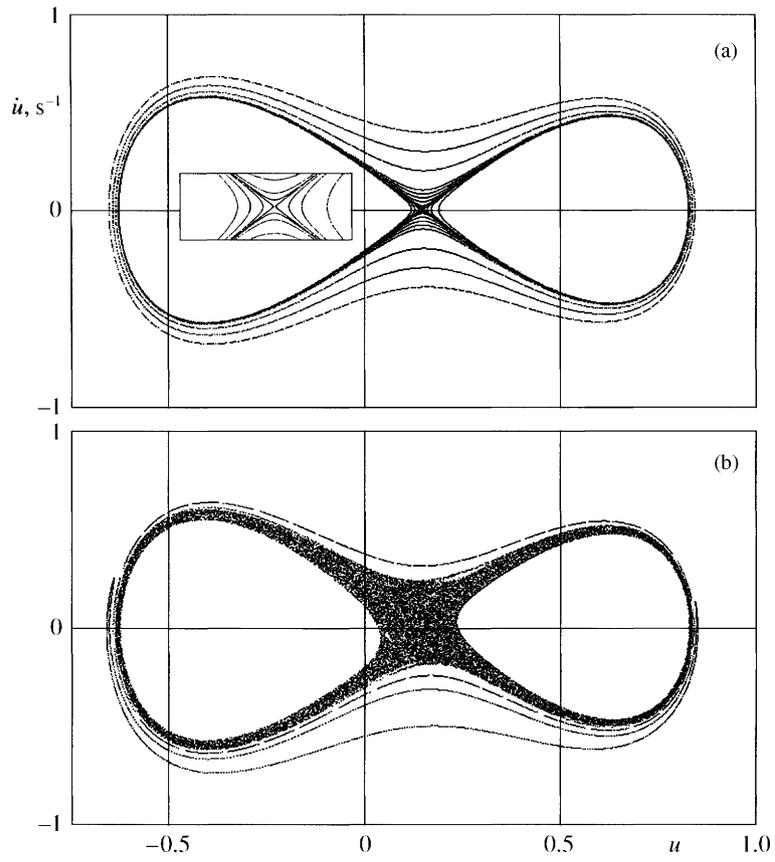


Fig. 3.

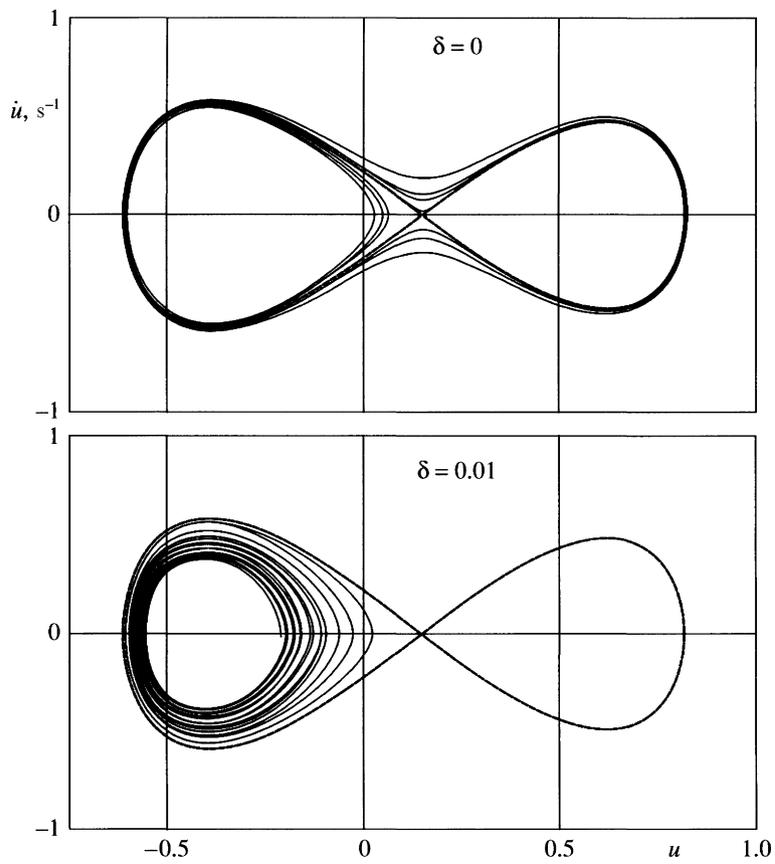


Fig. 4.

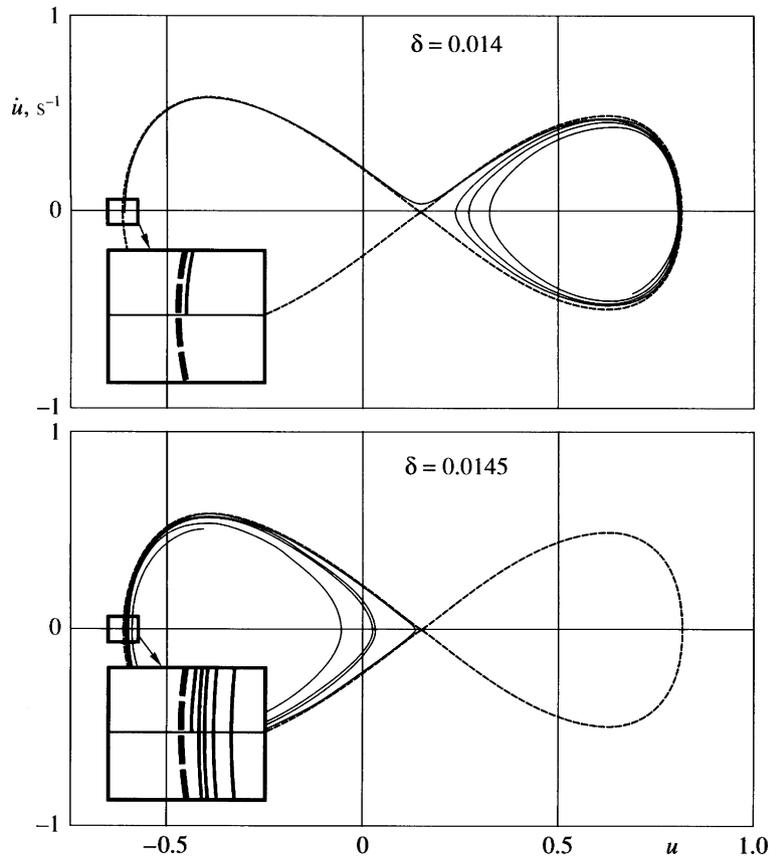


Fig. 5.

in the following cases: 1) the damping moment coefficient is less than the critical value for region A_2 but greater than the critical value for region A_1 : $\delta_1^* < \delta = 0.014 < \delta_2^*$; the trajectory then leaves the region A_2 and transfers to the region A_1 , in which it is attracted to the corresponding centre (the upper part of Fig. 5), 2) the damping moment coefficient is greater than the critical value for region A_2 : $0.0145 = \delta > \delta_2^*$; then the trajectory does not leave the region A_2 in which the motion began (the lower part). Modelling with other initial conditions also confirms the high accuracy of the criteria obtained using Mel'nikov's method.

6. Conclusion

We have shown that chaotic behaviour of a spacecraft is possible when it moves around a centre of mass when descending through a planet's atmosphere. Mel'nikov's method has been used to obtain the criteria for the occurrence of chaos, which agree well with the results of computer simulation. It has been established that Mel'nikov's method enables us to determine a measure of the damping required to prevent transients, which, in practical problems, are undesirable, since they lead to unpredictable behaviour of the spacecraft. It should be noted that the measure of the damping (5.3), which can ensure that an appropriate choice is made of the shape of the spacecraft for set of parameters (5.1) in the range of large angles of attack A_2 ($\arccos u_2 = \alpha_2 < \alpha < \alpha_0 = \arccos u_0$) is half the value required in the region of small angles of attack A_1 ($\arccos u_0 = \alpha_0 < \alpha < \alpha_1 = \arccos u_1$). This fact indirectly indicates that, other conditions being equal, the spacecraft will tend to fall in the region of small angles of attack.

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