The Oscillations of a Spacecraft under the Action of the Tether Tension Moment and the Gravitational Moment

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Abstract. The motion about a center of mass of a spacecraft with a tethered system, designed to launch a re-entry capsule from an orbit is considered. In the deployment of the tethered system the direction and value of the tensile strength of the tether vary and, if the point of application of the tensile strength does not coincide with the mass center of the spacecraft, a moment occurs which leads to oscillations of the body with variable amplitude and frequency. A non-linear equation of the perturbed motion of the body about of the mass center under the action of the moment of the tether tension and the gravitational moment is derived. Assuming that the change in the value and direction of the tensile force is slow and the gravitational moment is equal to zero, approximate and exact solutions of the non-linear differential equations of the perturbed and the unperturbed motions are obtained in terms of elementary functions and elliptic Jacobi functions. Similar solutions in linear statement of problem for the case when the gravitational moment takes place are found.

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FORMULATOIN OF THE PROBLEM

In the majority of publications devoted to an analysis of space tethered systems, the object of the investigation is the tether and the load, in which the satellite is regarded as a point mass [1-8]. And only in the papers [9, 10] motion of a spacecraft relative of the mass center was considered. In this paper we assume that the law of variation of the tensile force of the tether and the trajectory of the load, attached to the tether, are known, and we investigate the oscillations of the satellite as a rigid body under the action of the tensile force of the tether and the gravitational moment. If the spacecraft is the extended body along an axis of symmetry, then the gravitational moment tend to set up the spacecraft along a local vertical, but the tensile strength moment tend to set up the spacecraft along a tether. Consider the motion of a spacecraft about of a mass center during the dynamic deployment of a tethered system with a re-entry capsule. In dynamic deployment the tether is released more rapidly than in static deployment [2, 4] and, under the action of the Coriolis force, the capsule is deflected from the vertical, and then, after the tether unfolds to its complete length, return motion of the capsule to the vertical begins. The tether tension, variable in value and direction, produces an additional moment, under the action of which the satellite performs non-stationary oscillations about of the mass center, which, in turn, leads, for example, to the occurrence of an undesirable additional microaccelerations. The gravitational and Coriolis forces, which lie in the orbital plane of the spacecraft, have a decisive effect on the motion of the tethered system, and hence it is completely justified to consider the plane motion of the tethered system and the body. The aim of this paper is to obtain approximate and exact solutions of the equations, which describe the perturbed and unperturbed motion about of the mass center of a spacecraft with a tethered system.

THE EQUATION OF PERTURBED MOTION

Consider a mechanical system (Fig. 1), consisting of the spacecraft with of the mass center at the point O, a tether P_1P_2 and a re-entry capsule P_2. The spacecraft moves in a elliptical orbit (represented in Fig. 1 by the dashed curve). We introduce in the orbital plane a system of coordinates Ox_1y_1, in which the Ox_1 axis coincides with the local vertical, and a system of coordinates $Oxyz$ connected with the satellite, in which the Oxy plane coincides with the orbital plane and the Ox axis is directed along the longitudinal axis of the spacecraft. To derive the equation describing the motion of the satellite about the centre of mass, we will use the theorem of the change in the angular momentum [11] projected onto the Oz axis, perpendicular to the plane of motion Oxy and the equation motion of a material point in the central field [12]. We obtain

$$C(\ddot{\alpha}+\ddot{\vartheta}) = -3n^2(1-e^2)^{-3}(1+e\cos\vartheta)^3(B-A)\sin\alpha\cos\alpha - T\Delta\sin(\alpha-\varphi), \quad (1)$$
$$\dot{\vartheta} = n(1-e^2)^{-3/2}(1+e\cos\vartheta)^2 .$$

Here α is the angle between the longitudinal axis of the body and the local vertical, $A \le B < C$ are the principal components of the inertia tensor of the body in the coupled system of coordinates $Oxyz$, T is the value of the tether tension, e is orbit eccentricity, ϑ is the orbit true anomaly, φ is the angle between the line of action of the tensile force of the tether and the local variable, $\Delta = OP_1$ (Fig.1) and $n = \sqrt{\gamma M / r_3^3}$, where γ is the universal gravitational constant, M is the mass of the Earth, and r_3 is the distance from the body to the centre of the Earth.

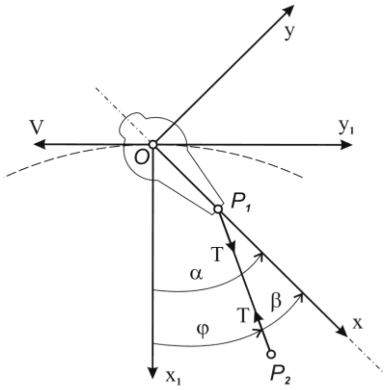

FIGURE 1. This is the Style for Figure Captions. Center this text if it doesn't run for more than one line.

Suppose the tether tension and the angle between the line of action of the tether tension and the vertical are slowly varying functions of the slow time $\tau = \varepsilon t$, where $\varepsilon = 0$ is a small parameter. We will consider the case, when spacecraft moves on a circular orbit ($e = 0$), after which equation (1) takes the form

$$\ddot{\alpha} + a(\tau)\sin\alpha - b(\tau)\cos\alpha + c\sin\alpha\cos\alpha = 0 \quad (2)$$

where $a(\tau) = T(\tau)\Delta C^{-1}\cos\varphi(\tau) > 0, \quad b(\tau) = T(\tau)\Delta C^{-1}\sin\varphi(\tau), \quad c = 3n^2\left(B-A\right)C^{-1} \ge 0.$ (3)

DYNAMICALLY SYMMETRIC SPACECRAFT

Let's consider a case, when the moments of inertia take $A = B$, then the equation (2) can written as

$$\ddot{\alpha} + a(\tau)\sin\alpha - b(\tau)\cos\alpha = 0 . \quad (4)$$

If $\varepsilon = 0$ we obtain the equation of a unperturbed motion, and it allows of a first integral, namely, the energy integral

$$\frac{\dot{\alpha}^2}{2} - a\cos\alpha - b\sin\alpha = H = const . \quad (5)$$

We obtain a solution of the equation of unperturbed motion by considering only the oscillatory motion between two positions: $\alpha_1 = \alpha_{\min}$ and $\alpha_2 = \alpha_{\max}$. We will choose the following initial conditions $t = 0$: $\alpha_0 = \alpha_2$, $\dot{\alpha}_0 = 0$. Separating the variables in equation (5), we obtain

$$t = -\int_{\alpha_2}^{\alpha} \frac{d\alpha}{\sqrt{2(H + b\sin\alpha + c\cos\alpha)}} . \quad (6)$$

Making two the changes of variables [13]

$$\alpha = 2\psi + \varphi, \quad \sin\psi = k\sin\gamma . \quad (7)$$

we convert the integral (6) to the incomplete elliptic integral of the first kind [14]

$$\omega t=-\int_{\pi/2}^{\gamma}\frac{d\gamma}{\sqrt{1-k^2\sin^2\gamma}},\quad \omega=\sqrt{T\frac{\Delta}{C}},\quad k^2=\sin^2\frac{\varphi-\alpha_2}{2}=\sin^2\frac{\varphi-\alpha_1}{2}. \tag{8}$$

Introducing the Jacobi function [14] we can write the generating decision of the equation (4) as follows:

$$\alpha=\varphi-2\arcsin sn(\omega t-K(k),k). \tag{9}$$

where $sn(u,k)$ is the elliptic sine, $K(k)$ is the complete elliptic integral of the first kind.

For a perturbed single-frequency system with slowly varying parameters (4), it is of interest to consider the action integral [15]. The action integral is an adiabatic invariant and we can be written in two forms

$$I(\tau)=2\int_{\alpha_1}^{\alpha_2}\dot\alpha d\alpha=const,\quad I(\tau)=\int_{t}^{t+T_\alpha}\dot\alpha^2 dt=const. \tag{10}$$

where T_α is the period of the oscillations of the angle α.

We use the first form for the action integral (10) and we can write similarly to the formula (6)

$$I(\tau)=2\int_{\alpha_1}^{\alpha_2}\dot\alpha d\alpha=2\int_{\alpha_1}^{\alpha_2}\sqrt{2(H+b\sin\alpha+c\cos\alpha)}\,d\alpha=const.$$

Taking the replacements of variables (7), we obtain

$$I=16\omega\int_0^{\pi/2}\left(1-k^2\sin^2\gamma\right)^{-1/2}\cos^2\gamma d\gamma=16\omega\left[E(k)-(1-k^2)K(k)\right]=const. \tag{11}$$

where $E(k)$ is the complete elliptic integral of the second kind.

For the perturbed motion, the value of the tensile force of the tether and its direction are determined by the known slowly varying functions $T=T(\tau)$ and $\varphi=\varphi(\tau)$. It is obvious that the amplitude values of the angle of deflection of the body from the vertical will also change: $\alpha_1=\alpha_{\min}$ and $\alpha_2=\alpha_{\max}$. We apply to the method described in [16]. Using expansion into a series the complete elliptic integrals of the first and second kind in power series and inversion of series in the formula (11), we write the following expression for the minimum and maximum angle of deflection of the spacecraft from the vertical in the following form

$$\alpha_{1,2}=\varphi(\tau)\mp 2\arcsin\left\{\frac{D}{\omega(\tau)}-\frac{1}{2\pi}[\frac{D}{\omega(\tau)}]^2-\frac{1}{4\pi^2}[\frac{D}{\omega(\tau)}]^3-...\right\}^{1/2}. \tag{12}$$

where $D=const$.

DYNAMICALLY ASYMMETRIC SPACECRAFT

Now we shall consider dynamically asymmetrical a spacecraft $A\neq B$ ($A<B$). Let's believe, that the angle of a deviation of the longitudinal axis of the spacecraft from a local vertical is small ($\sin\alpha\approx\alpha,\ cos\alpha\approx 1$) then the equation (2) can be written as

$$\ddot\alpha+\left[a(\tau)+c\right]\alpha-b(\tau)=0. \tag{13}$$

We make transformations similar (5) – (7) and we write the following formula for $\varepsilon=0$

$$t=-\int_{\alpha_2}^{\alpha}\frac{d\alpha}{\sqrt{-(a+c)\alpha^2+2b\alpha+2h}}=\frac{1}{\omega}\arcsin\left(\frac{b-\omega^2\alpha}{\sqrt{b^2+2h\omega^2}}\right)\Bigg|_{\alpha_2}^{\alpha},$$

where $\omega=\sqrt{a+c}$, $h=\omega^2\alpha_2^2/2-b\alpha_2$.

The generating decision ($\varepsilon=0$) of the equation (13) is given by

$$\alpha(t)=\frac{b}{\omega^2}+\left(\alpha_2-\frac{b}{\omega^2}\right)cos(\omega t). \tag{14}$$

For the perturbed motion we use the second form for the action integral (10) and, employing the decision (14), we can write the action integral as

$$I = \omega^2 \left(\alpha_2 - \frac{b}{\omega^2} \right)^2 \int_0^{T_\alpha} \sin^2(\omega t)dt = \pi\omega \left(\alpha_2 - \frac{b}{\omega^2} \right) = const , \quad (15)$$

where $T_\alpha = 2\pi / \omega$, $\alpha_2 = \alpha_{\max}$.

Using the formulas (3) and (15) we can write the maximum angle of deflection of the spacecraft from the vertical as function of the tether tension and of the deviation angle

$$\alpha_{\max}(\tau) = \frac{const}{\sqrt{T(\tau)\Delta\cos\varphi(\tau) + 3n^2(B-A)}} + \frac{T(\tau)\Delta\sin\varphi(\tau)}{T(\tau)\Delta\cos\varphi(\tau) + 3n^2(B-A)} . \quad (16)$$

The minimal angle is defined integral of energy of the equation (13) at $\dot{\alpha} = 0$ for the unperturbed motion ($\varepsilon = 0$)

$$E(\alpha, \dot{\alpha} = 0) = (a+c)\alpha_{\max}^2 / 2 - b\alpha_{\max} = (a+c)\alpha_{\min}^2 / 2 - b\alpha_{\min} = const .$$

CONCLUSION

In this paper the approximate and exact solutions of the equations, which describe the perturbed and unperturbed motion about of the mass center of a spacecraft with a tethered system, is obtained. These decisions can be used for definition an undesirable additional microaccelerations, which arise at fluctuations of a spacecraft under action the tether tension.

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