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The effect of the elasticity of an orbital tether system on the oscillations of a satellite☆

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ABSTRACT

An orbital tether system, including a satellite (a rigid body), an elastic ponderable tether and a terminal load, is investigated. A mathematical model is obtained using Lagrange's equation of the second kind, which enables the plane translational motion of the centres of mass of the elements of the system and the rotational motion of the satellite and the tether to be investigated. It is shown that the equations of motion for the new independent variable, that is, the true anomaly angle, obtained on the assumption that the motion of the centre of mass of the system is independent of the relative motion of its elements, are an extension of the known mathematical models. The effect of the elasticity of the tether on the angular oscillations of the tether and the satellite is investigated. The model constructed can be used both to analyse of the deployment of a tether system as well as to investigate of the combined behaviour of a satellite and a tether about the natural centres of mass.

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In the majority of known mathematical models of orbital tether systems,[1] the satellite is considered as a point mass that does not enable them to be used to analyse the motion of the satellite itself as a rigid body. Special mathematical models have been used in attempts to investigate the effect of the tether on the behaviour of a satellite,[2–4] but these do not enable one to solve the problem of the influence of elastic and other forces on the behaviour of the tether and the satellite in a more general formulation.

1. Formulation of the problem

We shall assume that the satellite is a rigid body on which a gravitational force, a gravitational moment and a moment due to the tensile force of the tether act. In a dynamic sense, the satellite is defined by the principal moments of inertia and the centre of mass, and the point of the tether deployment does not coincide with the centre of mass of the satellite.

In deriving the equations of motion, we shall neglect the flexibility of the elastic tether, the mass of which changes in direct proportion to its length. We shall represent the terminal load as a point mass.

In order to construct a mathematical model, we shall use Lagrange's equations of the second kind, taking account of both potential as well as non-potential forces. We shall investigate the motion of the system in the orbital plane.

We will solve several problems. Initially, without introducing constraints on the ratio of the geometrical dimensions of the satellite and the tether and assuming the elastic tether to be ponderable, we will obtain the equations of motion of the tether system that are not explicitly solved with respect to the generalized coordinates. The equations of motion, which are explicitly solved with respect to the generalized coordinates, will be constructed assuming that the length of the tether is considerably greater than the geometrical dimensions of the satellite and that the ponderable elastic tether is fully deployed. Assuming the motion of the centre of mass of the system to be independent of the motion of its elements, the equations of motion will be written for the new independent variable, that is, the true anomaly angle, which enables us to reduce the system of equations of motion to three second-order equations. It will be shown that the resulting equations generalize the previously known equations of motion. An approximate analytical solution will be found for the equation of the longitudinal oscillations of the tether in the case when the tether coincides with a local vertical. The effect of the elastic properties of the tether on the angular behaviour of the tether and the satellite will be investigated in the last part of this paper, and the chaotic character of this effect will be shown.

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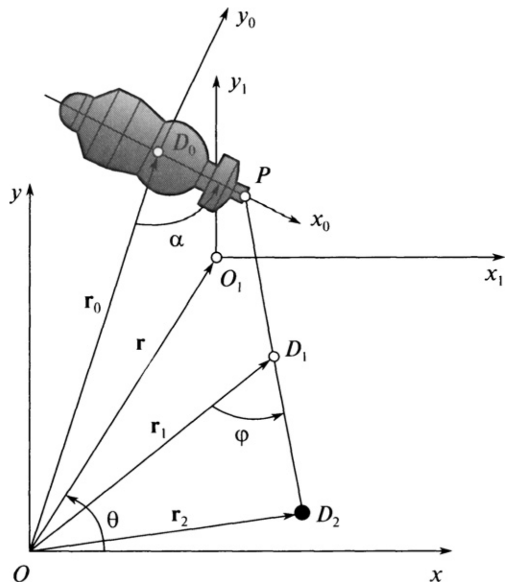

Fig. 1.

2. Lagrange's equations of the second kind

The tether system consists of a satellite of mass m_0 with its centre of mass at the point D_0, a tether of mass $m_1(l) = \rho Sl$ (l is the length, ρ is the density, and S is the cross-section area of the tether) with its centre of mass at the point D_1 and a terminal load of mass m_2 at the point D_2 (Fig. 1). The total mass of the system is

$$m = m_0 + m_1 + m_2 = m_0^0 + m_2 \tag{2.1}$$

where m_0^0 is the initial mass of the satellite and $m_0 = m_0^0 - m_1$ is the current mass of the satellite.

According to König=s theorem, the kinetic energy of the tether system consists of the kinetic energy of the centre of mass T_C and the kinetic energy of the bodies and point masses of the system in the translationally moving system of coordinates $O_1x_1y_1$ with its origin at the centre of mass (Fig. 1):

$$T = T_C + T_0 + T_1 + T_2 \tag{2.2}$$

where $T_i (i = 0, 1, 2)$ are the kinetic energies of the elements of the system in the system of coordinates $O_1x_1y_1$. We choose

$$q_1 = \alpha, \quad q_2 = \varphi, \quad q_3 = l, \quad q_4 = \theta, \quad q_5 = r \tag{2.3}$$

as the generalized coordinates, where $r = OO_\perp$ is the distance between the centre of the planet and the centre of mass of the system O_1 (Fig. 1), $l = PD_2$ is the tether length, θ is the true anomaly angle of the centre of mass of the system (the system of coordinates Oxy is located in the orbital plane of the orbit and the Ox axis is directed on the perigee), φ is the angle of deflection of the tether from the line OD_1 and α is the angle of deflection of the satellite axis from the line OD_0.

The kinetic energy of the centre of mass of the system is equal to

$$T_C = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) \tag{2.4}$$

The kinetic energies of the relative motion of the satellite, the tether and the load are defined respectively in the form

$$T_0 = \frac{1}{2}m_0V_{0r}^2 + \frac{1}{2}C\omega_0^2, \quad T_1 = \frac{1}{2}m_1V_{1r}^2 + \frac{1}{2}C_1\omega_1^2, \quad T_2 = \frac{1}{2}m_2V_{2r}^2 \tag{2.5}$$

where V_{ir} is the velocity of a point D_i in the system of coordinates $O_1x_1y_1 (i = 0, 1, 2)$, ω_0 and ω_1 are the angular velocities of the satellite and the tether, C is the principal moment of inertia of the satellite about the D_0z_0 axis in the system of coordinates $D_0x_0y_0z_0$ and $C_1 = m_1l^2/12$ is the moment of inertia of the tether about to its centre of mass. In order to determine the kinetic energy of the tethered system (2.2) in terms of the generalized coordinates (2.3), we will introduce the radius vectors of the centre of mass of the system O_1 and of the centres of mass of its individual elements D_i in polar coordinates

$$\mathbf{r} = \mathbf{OO}_1 = (r, \theta), \quad \mathbf{r}_1 = \mathbf{OD_i} = (r_i, \theta_i), \quad i = 0, 1, 2 \tag{2.6}$$

and, also, the vectors

$$\mathbf{l} = \mathbf{PD}_2 = (l, \varphi), \quad \mathbf{\Delta} = \mathbf{D_0P} = (\Delta, \alpha) \tag{2.7}$$

By virtue of the definition of the centre of mass and the geometrical arrangement of the points (Fig. 1), vectors (2.6) and (2.7) are connected by the relations

$$m\mathbf{r} = \sum_{i=0}^{2} m_i\mathbf{r_i}, \quad \mathbf{r}_t = \mathbf{r}_1 + \mathbf{\Delta} + \frac{1}{2}\mathbf{l}, \quad \mathbf{r}_2 = \mathbf{r}_1 + \mathbf{\Delta} + \mathbf{l}$$

The formulae establishing the link between the coordinates of the points D_i and the generalized coordinates (2.3)

$$\mathbf{r}_0 = \mathbf{r} - \frac{m_1+m_2}{m}\mathbf{\Delta} - \frac{m_1/2+m_2}{m}\mathbf{l}, \quad \mathbf{r}_1 = \mathbf{r} + \frac{m_0}{m}\mathbf{\Delta} + \frac{m_0+m_2}{2m}\mathbf{l} \tag{2.8}$$

$$\mathbf{r}_2 = \mathbf{r} + \frac{m_0}{m}\mathbf{\Delta} + \frac{m_0+m_1/2}{m}\mathbf{l} \tag{2.9}$$

follow from this. The position of the points D_i in the system of coordinates $O_1x_1y_1$ associated with the centre of mass of the tether system can be found from formulae (2.8) and (2.9):

$$\boldsymbol{\rho}_i = \mathbf{r}_i - \mathbf{r}, \quad i = 0, 1, 2$$

and, consequently, also the velocities of these points

$$\mathbf{V}_{ir} = \dot{\boldsymbol{\rho}}_i \tag{2.10}$$

The angular velocities of the satellite and the tether, occurring in the first two formulae of (2.5), are:

$$\omega_0 = \dot{\alpha} + \dot{\theta}_0, \quad \omega_1 = \dot{\varphi} + \dot{\theta}_1 \tag{2.11}$$

In their turn, the derivatives $\dot{\theta}_0$ and $\dot{\theta}_1$ can be obtained using the projections of the vectors 92.8) on to the axes of the $O_1x_1y_1$ system (Fig. 1). We have

$$r_0\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}\theta_0 = r\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}\theta - \frac{m_1+m_2}{m}\Delta\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}(\theta_0+\alpha) - \frac{m_1/2+m_2}{m}l\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}(\theta_1-\varphi)$$

$$r_1\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}\theta_1 = r\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}\theta + \frac{m_0}{m}\Delta\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}(\theta_0+\alpha) + \frac{m_0-m_2}{2m}l\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}(\theta_1+\varphi)$$

Finally, using formulae (2.4), (2.5), (2.10) and (2.11), we write the kinetic energy of the tether system (2.2) as a function of generalized coordinates (2.3) and their velocities in the form

$$T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{1}{2}\sum_{i=0}^{2} m_i\dot{\rho}_i^2 + \frac{1}{2}[C_0(\dot{\alpha}+\dot{\theta}_0)^2 + C_1(\dot{\varphi}+\dot{\theta}_1)^2] \tag{2.12}$$

The potential energy of the system is equal to the sum of the potentials of the central field of the gravitational force[5] and the potential energy of the elastic force of the tether W_E

$$W_0 = -\frac{\mu m_0}{r_0} + \frac{3\mu}{2r_0^3}(A-B)\cos^2\alpha, \quad W_1 = -\frac{\mu m_1}{r_1}\left(1 + \frac{l^2}{8r_1^2}\cos^2\varphi\right), \quad W_2 = -\frac{\mu m_2}{r_2}$$

$$W_E = \frac{c}{2}(l-l_0)^2$$

where μ is a gravitational parameter, A and B are the principal moments of inertia about the D_0x_0 and D_0y_0 axes in the coordinate system $D_0x_0y_0z_0$ associated with the satellite, $c = E/l_0$ is the coefficient of elasticity, l_0 is the length of the unstressed tether and E is the modulus of elasticity. Combining these expressions, we obtain the formula for the potential energy of the system

$$W = -\mu\sum_{i=0}^{2}\frac{m_i}{r_i} + \frac{3\mu}{2r_0^3}(A-B)\cos^2\alpha - \frac{\mu m_1 l^2}{8r_1^3}\cos^2\varphi + \frac{c}{2}(l-l_0)^2 \tag{2.13}$$

Taking account of relations (2.12) and (2.13), we write the expression for the Lagrangian

$$L = T - W = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{1}{2}\sum_{i=0}^{2} m_i\dot{\rho}_i^2 + \frac{1}{2}[C_0(\dot{\alpha} + \dot{\theta}_0)^2 + C_1(\dot{\varphi} + \dot{\theta}_1)^2]$$

$$+ \mu\sum_{i=0}^{2}\frac{m_i}{r_i} - \frac{3\mu}{2r_0^3}(A - B)\cos^2\alpha + \frac{\mu m_1 l^2}{8r_1^3}\cos^2\varphi - \frac{c}{2}(l - l_0)^2 \quad (2.14)$$

Lagrange's equations of the second kind have the form

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = Q_j, \quad j = 1, 2, \ldots, 5 \quad (2.15)$$

where $q_1 = \alpha$, $q_2 = \varphi$, $q_3 = l$, $q_4 = \theta$, $q_5 = r$, Q_j are non-potential forces, including control, damping, aerodynamic and other forces.

Note that Lagrangian (2.14) depends on the generalized coordinates q_j and the generalized velocities $\dot{q}_j$ by virtue of relations (2.4)–(2.13), albeit implicitly.

3. Lagrange's equations of the second kind for a deployed tether system

Lagrangian (2.14) can be considerably simplified if certain assumptions are introduced. First, suppose the tether is fully deployed. Second, the tether length l is much greater than the stretching force arm of the tether Δ:

$$\Delta/l \ll 1 \quad (3.1)$$

(in the experiment YES2,[1] the tether had a length l of the order of 30 km and the stretching force arm Δ had a length of a few metres). Third, the tether length is many times shorter than the distance from the centre of mass of the system to the centre of the Earth, and we shall therefore assume that $\theta_0 = \theta_1 = \theta$. Fourth, we shall neglect the mass of the tether. In this case, the mass of the tether system is given by the relation

$$m = m_0 + m_2 \quad (3.2)$$

where m_0 and m_2 are constant quantities.

By virtue of the ratio (3.1), it can be assumed that the centre of mass of the whole system lies on the line PD_2 (Fig. 1). In order to determine the kinetic energy of the satellite T_0 (the first equality of (2.5)), we first find the projections of the point D_0 in the translationally moving system of coordinates $O_1x_1y_1$

$$x_0 = \bar{m}_2 l\cos(\theta + \varphi) + \Delta\cos(\theta + \alpha),$$

$$y_0 = \bar{m}_2 l\sin(\theta + \varphi) + \Delta\sin(\theta + \alpha); \quad \bar{m}_2 = m_2/m \quad (3.3)$$

It is obvious that the velocity of the point D_0 in the system of coordinates $O_1x_1y_1$ is equal to

$$V_{0r} = (\dot{x}_0^2 + \dot{y}_0^2)^{1/2} \quad (3.4)$$

Substituting the time derivatives of the coordinates (3.3) into equality (3.4) and also keeping the first relation of (2.11) in mind, we can write the expression for the kinetic energy of the satellite (the first equality of (2.5)) as follows:

$$T_0 = \frac{m_0}{2}[\bar{m}_2^2\dot{l}^2 + \bar{m}_2^2 l^2(\dot{\varphi} + \dot{\theta})^2 + 2\Delta\bar{m}_2\dot{l}(\dot{\alpha} + \dot{\theta})\sin(\varphi - \alpha)$$

$$+ 2\Delta\bar{m}_2 l(\dot{\varphi} + \dot{\theta})(\dot{\alpha} + \dot{\theta})\cos(\varphi - \alpha)] + \frac{C_0}{2}(\dot{\alpha} + \dot{\theta})^2; \quad C_0 = C + m_0\Delta^2 \quad (3.5)$$

Taking account of the fact that the distance between the centre of mass of the system and the point D_0 is equal to $\bar{m}_0 l$, for the kinetic energy of the relative motion of the terminal load D_2, which is given by the last equality of (2.5), we obtain

$$T_2 = \frac{m_2\bar{m}_0^2}{2}[l^2(\dot{\varphi} + \dot{\theta})^2 + \dot{l}^2] \quad (3.6)$$

Finally, using expressions (2.2), (2.4), (3.5) and (3.6), we can write the kinetic energy of the system as follows:

$$T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{m_*}{2}\dot{l}^2 + \frac{I_*}{2}(\dot{\varphi} + \dot{\theta})^2 + \frac{C_0}{2}(\dot{\alpha} + \dot{\theta})^2$$

$$+ \Delta m_*[\dot{l}\sin(\varphi - \alpha) + l(\dot{\varphi} + \dot{\theta})\cos(\varphi - \alpha)](\dot{\alpha} + \dot{\theta}); \quad m_* = \frac{m_0 m_2}{m}, \quad I_* = m_* l^2$$

By virtue of the assumptions made in this section, the potential energy (2.13) takes the form

$$W = -\frac{\mu m}{r} + \frac{\mu I_*}{2r^3}(1-3\cos^2\varphi) + \frac{3\mu}{2r^3}(A-B)\cos^2\alpha + \frac{c}{2}(l-l_0)^2 \tag{3.7}$$

By virtue of Eqs (2.5), the following equations of motion then correspond to the Lagrangian $L = T - W$

$$\begin{aligned}
&C_0\ddot\alpha + [C_0 + \Delta m_* l\cos(\varphi-\alpha)]\ddot\theta + \Delta m_* l\cos(\varphi-\alpha)\ddot\varphi \\
&+ \Delta m_*\sin(\varphi-\alpha)\ddot l + 2\Delta m_*\dot l(\dot\varphi+\dot\theta)\cos(\varphi-\alpha) \\
&- \Delta m_* l(\dot\varphi+\dot\theta)^2\sin(\varphi-\alpha) - \frac{3\mu}{r^3}(A-B)\sin\alpha\cos\alpha = Q_\alpha \\
&\Delta m_* l\cos(\varphi-\alpha)\ddot\alpha + [I_* + \Delta m_* l\cos(\varphi-\alpha)]\ddot\theta + 2m_* l\dot l(\dot\varphi+\dot\theta) + I_*\ddot\varphi \\
&+ \Delta m_* l(\dot\alpha+\dot\theta)^2\sin(\varphi-\alpha) + \frac{3\mu I_*}{r^3}\sin\varphi\cos\varphi = Q_\varphi \\
&\Delta\sin(\varphi-\alpha)(\ddot\alpha+\ddot\theta) + \ddot l + \frac{c}{m_*}(l-l_0) + \frac{\mu l}{r^3}(1-3\cos^2\varphi) \\
&- l(\dot\varphi+\dot\theta)^2 - \Delta(\dot\alpha+\dot\theta)^2\cos(\varphi-\alpha) = \frac{Q_l}{m_*} \\
&(mr^2 + C_0 + I_*)\ddot\theta + C_0\ddot\alpha + I_*\ddot\varphi + 2mr\dot r\dot\theta + 2m_* l\dot l(\dot\theta+\dot\varphi) = Q_\theta \\
&\ddot r - r\dot\theta^2 + \frac{\mu}{r^2} - \frac{3\mu I_*}{2mr^4}(1-3\cos^2\varphi) - \frac{9\mu}{2mr^4}(A-B)\cos^2\alpha = Q_r
\end{aligned} \tag{3.8}$$

We now use the technique[6–8] known for this problem and, in the equations of motion (3.8), change to the new independent variable, that is, the true anomaly angle θ. We shall assume here that the motion of the centre of mass of the tether system is independent of its relative motion and the centre of mass moves along an elliptic trajectory:

$$r = \frac{p}{k}, \quad \dot\theta = nk^2, \quad k = 1 + e\cos\theta, \quad n = \sqrt{\mu p^{-3}} \tag{3.9}$$

where p is the parameter of the orbit and e is its ellipticity. By virtue of relations (3.9), the first and second derivatives of an arbitrary function $f(t)$ have the form

$$\dot f = nk^2 f', \quad \ddot f = n^2k^3(kf'' - 2ef'\sin\theta) \tag{3.10}$$

A prime denotes differentiation with respect to θ. Note that differentiation of the second equality of (3.9) with respect to time gives

$$\ddot\theta = -2n^2k^3e\sin\theta \tag{3.11}$$

Taking account of formulae (3.9)–(3.11), we can rewrite the equations for the relative motion of the tether system (3.8) as follows:

$$\begin{aligned}
&C_0(k\alpha'' - 2e\alpha'\sin\theta) + \Delta m_* l\cos(\varphi-\alpha)(k\varphi'' - 2e\varphi'\sin\theta) \\
&+ \Delta m_*\sin(\varphi-\alpha)(kl'' - 2el'\sin\theta) + 2k\Delta m_* l'(1+\varphi')\cos(\varphi-\alpha) \\
&- k\Delta m_* l(1+\varphi')^2\sin(\varphi-\alpha) - 2e[C_0 + \Delta m_* l\cos(\varphi-\alpha)]\sin\theta \\
&- 3(A-B)\sin\alpha\cos\alpha = -\frac{Q_\alpha}{n^2k^3}
\end{aligned} \tag{3.12}$$

$$\begin{aligned}
&\Delta m_* l\cos(\varphi-\alpha)(k\alpha'' - 2e\alpha'\sin\theta) + I_*(k\varphi'' - 2e\varphi'\sin\theta) \\
&+ 2km_* ll'(1+\varphi') + \Delta km_* l(1+\alpha')^2\sin(\varphi-\alpha) \\
&- 2e[I_* + \Delta m_* l\cos(\varphi-\alpha)]\sin\theta + 3I_*\sin\varphi\cos\varphi = \frac{Q_\varphi}{n^2k^3}
\end{aligned} \tag{3.13}$$

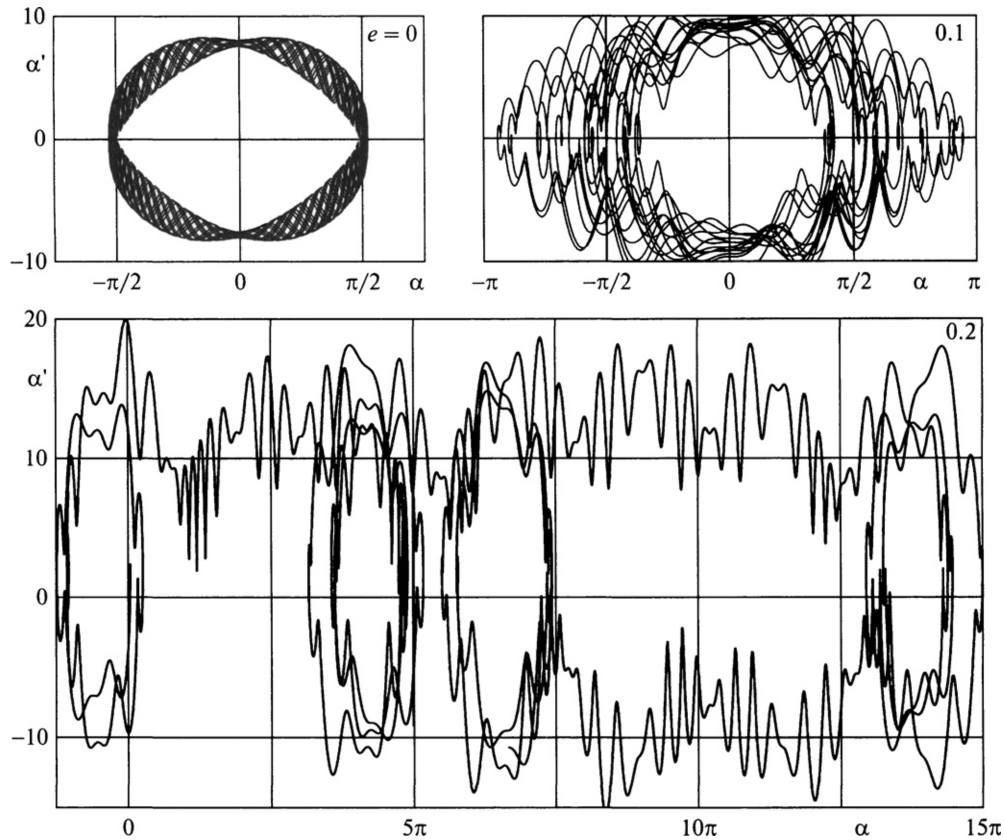

Fig. 2.

$$\Delta\sin(\varphi-\alpha)[k\alpha''-2e(1+\alpha')\sin\theta]+kl''-2el'\sin\theta+\frac{c(l-l_0)}{n^2k^3m_*}$$
$$+l(1-3\cos^2\varphi)-kl(1+\varphi')^2-\Delta k(1+\alpha')^2\cos(\varphi-\alpha)=\frac{Q_l}{m_*n^2k^3} \quad (3.14)$$

These equations are considerably simplified in the case of a circular orbit ($e=0$, $k=1$).

4. Elastic oscillations of a tether system

We shall assume that the point P, where the deployment of the tether starts, coincides with the centre of mass of the satellite ($\Delta=0$) and that the generalized force $Q_\varphi=0$. Equations (3.13) and (3.14) take the form

$$\varphi''+2\frac{l'}{l}(1+\varphi')+\frac{3}{k}\sin\varphi\cos\varphi = 2\frac{e}{k}(1+\varphi')\sin\theta \quad (4.1)$$

$$l''+\frac{c}{m_*n^2k^4}(l-l_0)+\frac{l}{k}(1-3\cos^2\varphi)-l(1+\varphi')^2 = 2\frac{e}{k}l'\sin\theta \quad (4.2)$$

For small e, chaos will be observed in the neighbourhood of the separatrix in the case of an inextensible tether.[8] On the other hand, an elastic tether also causes chaos in the case of the motion of a satellite in a circular orbit.[4] We note that Eq. (4.1) is an extension of the equation obtained earlier for an elastic tether (Ref. 7, Eq. (39))).

5. Oscillation's of a satellite with a vertical elastic tether

Suppose the centre of mass of the tether system moves in an elliptic orbit with a small eccentricity, the tether is deployed along a local vertical and its relative elongation is small. Then, taking account of condition (3.1), we shall assume that the following quantities are of the order of infinitesimals of ε

$$\delta = \Delta/l, \quad (l-l_0)\varphi/l_0,\ \varphi',\ \varphi'',\ e$$

(Condition A) and, when $Q_\alpha = Q_l = 0$, we rewrite Eqs (3.12) and (3.14), neglecting terms of the order of $O(\varepsilon^2)$ and higher. We obtain

$$\alpha'' - 3\frac{A-B}{kC_0}\sin\alpha\cos\alpha = -\delta J[(1-L'')\sin\alpha + 2L'\cos\alpha] + 2e(1+\alpha')\sin\theta = \varepsilon F_\alpha \tag{5.1}$$

$$L'' + \frac{c}{n^2k^4m_*}(L-1) - 3 = \delta[\sin\alpha\alpha'' + (1+\alpha')^2\cos\alpha] + 2e(\cos\theta - L'\sin\theta) = \varepsilon F_L \tag{5.2}$$

Here,

$$L = l/l_0,\ J = m_* l_0^2/C_0$$

We will now find an approximate law for the change in the length of the tether associated with its elasticity for which we consider the following equation, obtained from (5.2) when $O(\varepsilon)$ terms are discarded

$$L'' = \Omega^2 L = 3 + \Omega^2;\quad \Omega = \sqrt{c/m}/n \tag{5.3}$$

The equilibrium length of the tether is given by the formula $L_1 = (3+\Omega^2)/\Omega^2$.
For the initial conditions

$$t_0 = 0{:}\ L = L_1,\quad L' = L_0'$$

the solution of Eq. (5.3) has the form

$$L = L_1 + L_0'\sin(\Omega\theta)/\Omega \tag{5.4}$$

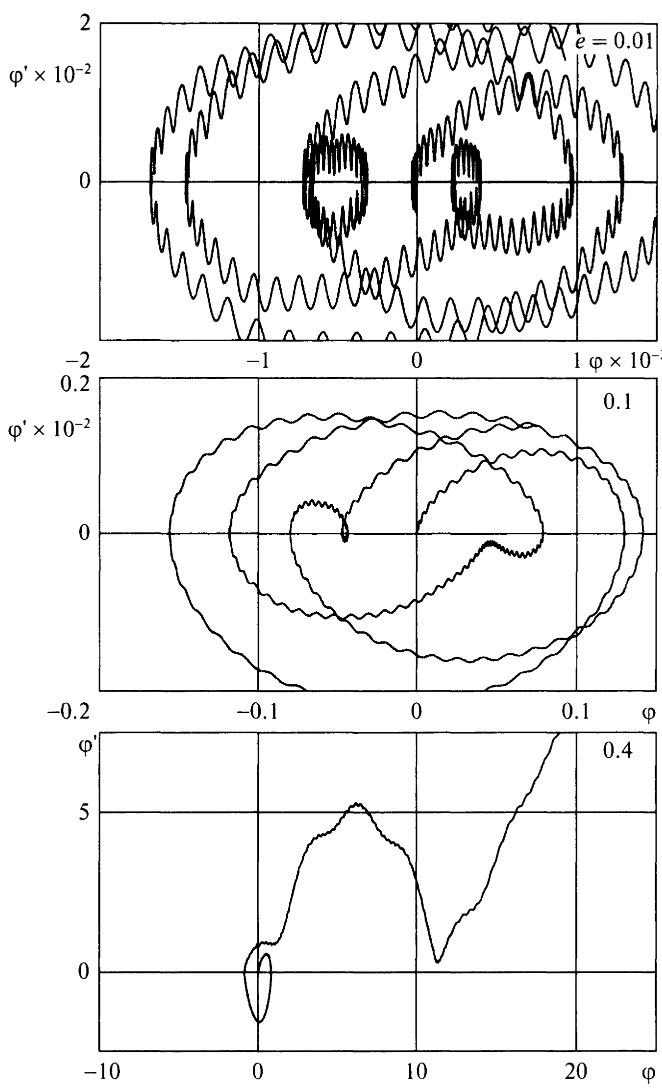

Fig. 3.

We shall investigate motions when the tether is always in the stretched out position ($L > 1$). The initial velocity of the load must then satisfy the condition

$$L'_0 < 3/\Omega$$

Substituting the corresponding derivatives of the solution (5.4) into the right-hand side of Eq. (5.1), we obtain the equation

$$\alpha'' - 3\frac{A-B}{C_0}\sin\alpha\cos\alpha = -\delta J[(1 + L'_0\Omega\sin\Omega\theta)\sin\alpha + 2L'_0\cos\Omega\theta\cos\alpha]$$

$$+ e\left[2(1+\alpha')\sin\theta - 3\frac{A-B}{C_0}\sin\alpha\cos\alpha\cos\theta\right] \equiv \varepsilon F_\alpha \quad (5.5)$$

The last term on the right-hand side of this equation defines the effect of the eccentricity on the angular motion of the satellite. The effect of the elasticity of the tether on this motion can be estimated separately by considering a circular orbit ($e = 0$).

6. Simulation of the motion of a satellite with an elastic tether

For the numerical simulation, we shall use the equations of motion of a satellite with an elastic tether (3.12)–(3.14). When Condition A is satisfied, Eq. (5.5), when $\varepsilon = 0$, can be used as the equation of the unperturbed motion:

$$\alpha'' + \lambda^2\sin\alpha\cos\alpha = 0; \quad \lambda^2 = -3\frac{A-B}{C_0} = 3\frac{B-A}{C + m_0\Delta^2} \quad (6.1)$$

We select the orbital parameter $p = 6.7 \times 10^3$ km, the satellite parameters $A = 10^3$ kg.m^2, $B = C = 10^5$ kg.m^2, $m_0 = 6 \times 10^3$ kg, $m_2 = 30$ kg, $\Delta = 30$ m and the tether parameters $l_0 = 20$ km, $d = 0.5$ mm, where d is the tether diameter. In the case of the above values of the parameters,

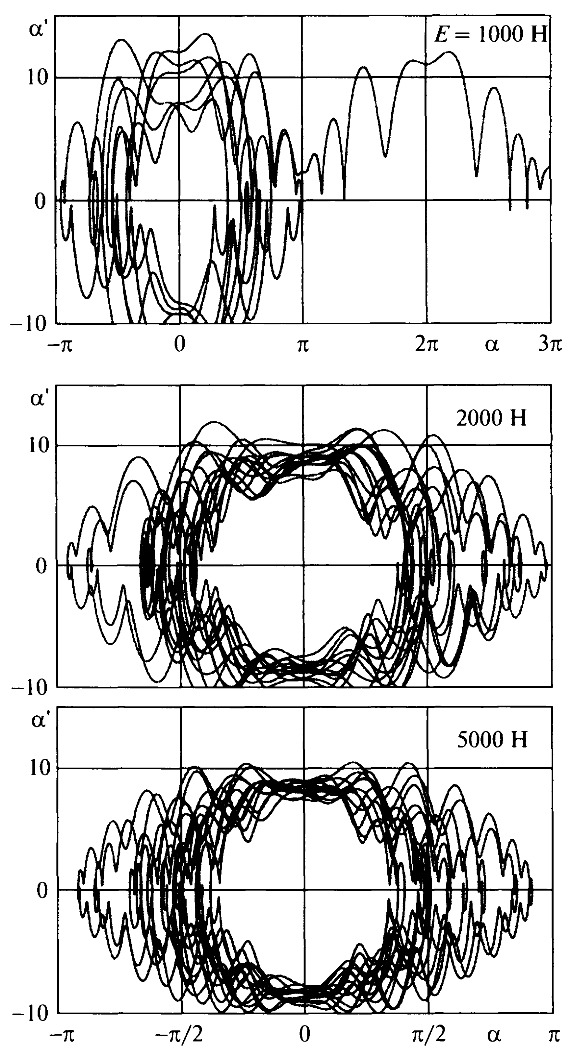

Fig. 4.

the natural frequency of the unperturbed system (6.1) $\lambda = 0.65$. The hyperbolic points of system (6.1) ($N = 0, 1, 2, \ldots$) correspond to an unstable equilibrium position. We choose the following initial conditions

$$\alpha = 1.57, \quad \alpha' = 0, \quad \varphi = 0, \quad \varphi' = 0, \quad l = l_0, \quad l' = 2.7 l_0/\Omega$$

The phase trajectories of the system $\alpha'(\alpha)$ and $\varphi'(\varphi)$ for $E = 3000$ N are shown in Figs. 2 and 3 for different values of the eccentricity. As e increases, an increase in the amplitude of the oscillations of the satellite and the tether is observed. For sufficiently large eccentricity values, the elements of the system tranfer into a rotational mode of motion (Fig. 2 when $e = 0.2$ and Fig. 3 when $e = 0.4$). In the case shown in Fig. 3 when $e = 0.4$, several transitions between rotational and oscillatory modes are observed which can be considered as a manifestation of the chaotic behaviour of the system in the neighbourhood of the separatrix.

The phase trajectories of the satellite $\alpha'(\alpha)$ are shown in Fig. 4 for different values of the modulus of elasticity of the tether E when $e = 0.1$. For comparatively small values of E, both oscillations as well as a rotation of the satellite are observed, (the upper part of Fig. 4). In the case of large values of E, only oscillations occur (the middle and lower parts of Fig. 4).

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